# Anderson Localization

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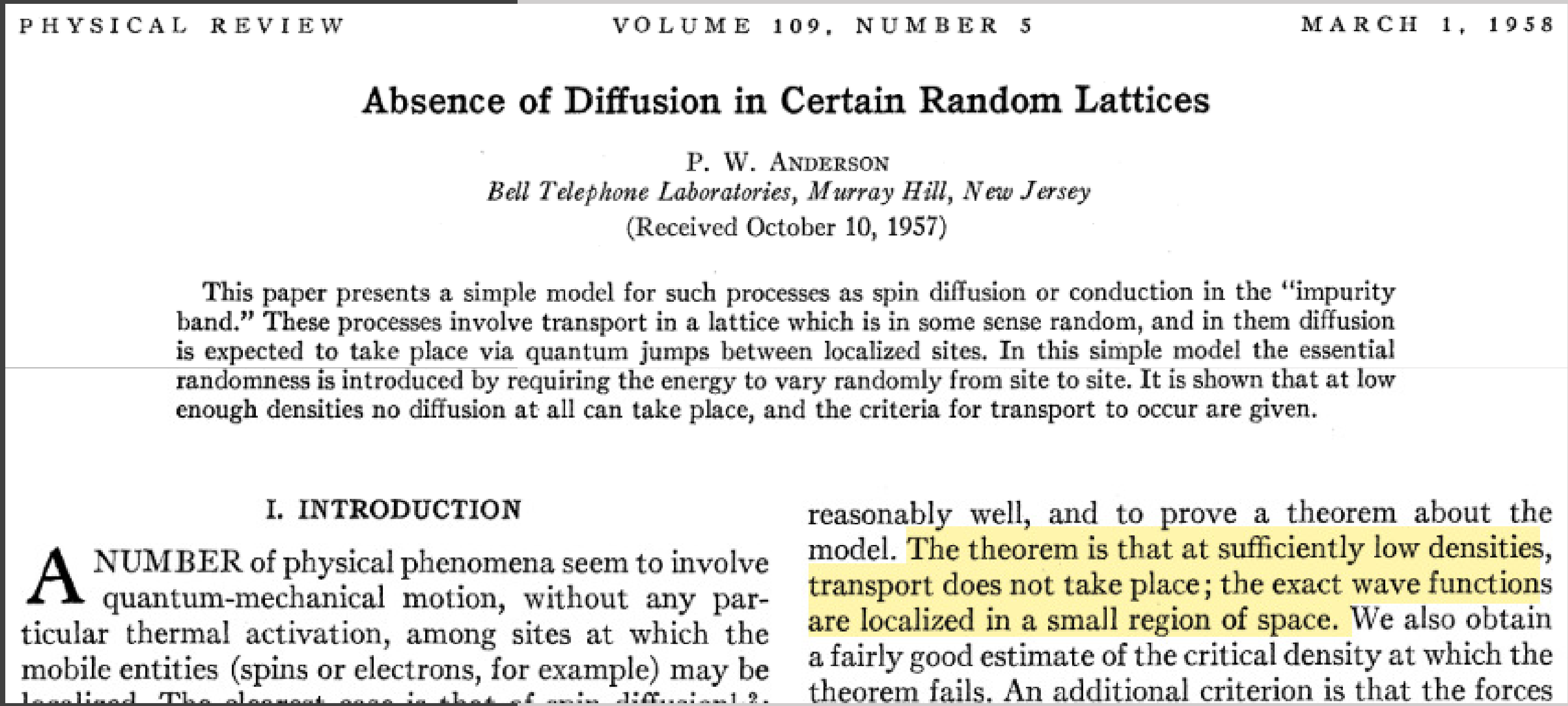
# Anderson

Phillip Warren Anderson
13/12/93 (94)
Harvard

★ ⅓ Nobel-1977
(⅓ Sir Neville Mott and
⅓ John Hasbrouk van Vleck)

“For the fundamental theoretical investigations of the electronic structure of magnetic and disordered systems”



PHYSICAL REVIEW VOLUME 109, NUMBER 5 MARCH 1, 1958

## Absence of Diffusion in Certain Random Lattices

P. W. ANDERSON
*Bell Telephone Laboratories, Murray Hill, New Jersey*
(Received October 10, 1957)

This paper presents a simple model for such processes as spin diffusion or conduction in the “impurity band.” These processes involve transport in a lattice which is in some sense random, and in them diffusion is expected to take place via quantum jumps between localized sites. In this simple model the essential randomness is introduced by requiring the energy to vary randomly from site to site. It is shown that at low enough densities no diffusion at all can take place, and the criteria for transport to occur are given.

### I. INTRODUCTION

A NUMBER of physical phenomena seem to involve quantum-mechanical motion, without any particular thermal activation, among sites at which the mobile entities (spins or electrons, for example) may be

reasonably well, and to prove a theorem about the model. The theorem is that at sufficiently low densities, transport does not take place; the exact wave functions are localized in a small region of space. We also obtain a fairly good estimate of the critical density at which the theorem fails. An additional criterion is that the forces

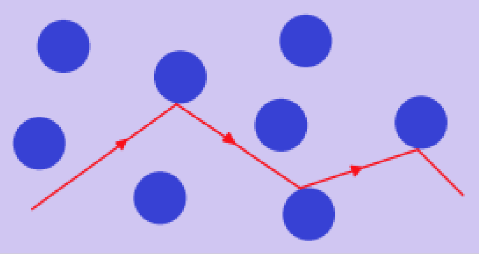
# Condução em metais

## Modelo classico

- Eletrons livres interagem com uma força “viscoça”
- τ é o tempo de relaxamento ou tempo medio entre colisões

$$\sigma = \frac{ne^2\tau}{m_e}$$

$$\ell = \bar{v}\tau \approx a$$

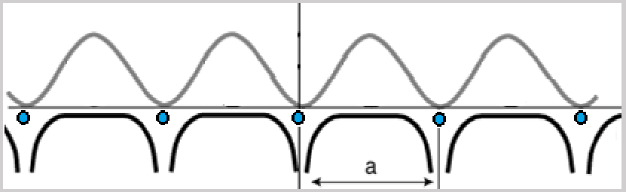


## Modelo quantico
(Drude - Sommerfeld, Landau, Bloch)

- + Fermi – Dirac statistics
- Densidade de estados (GAP)

$$\sigma = \frac{ne^2\tau}{m^*}$$

$$\ell = v_F\tau$$



# Localization

## Transição Metal-Isolante

### Mott–Hubbard transition

- Interações coulombicas

*e*-*e*   *e*-ion   *e*-impureza

A localização ocorre quando um eletron se enlaza com uma impureza

- Ligado a um atomo (GAP)

### Anderson transition

- Desordem
- Scattering

Muitos scatterings de uma onda podem aumentar os efeitos de interferencia até localizar espacialmente um eletron

- Confinamento numa região espacial

F. J. Morin Phys. Rev. Lett. **3**, 34

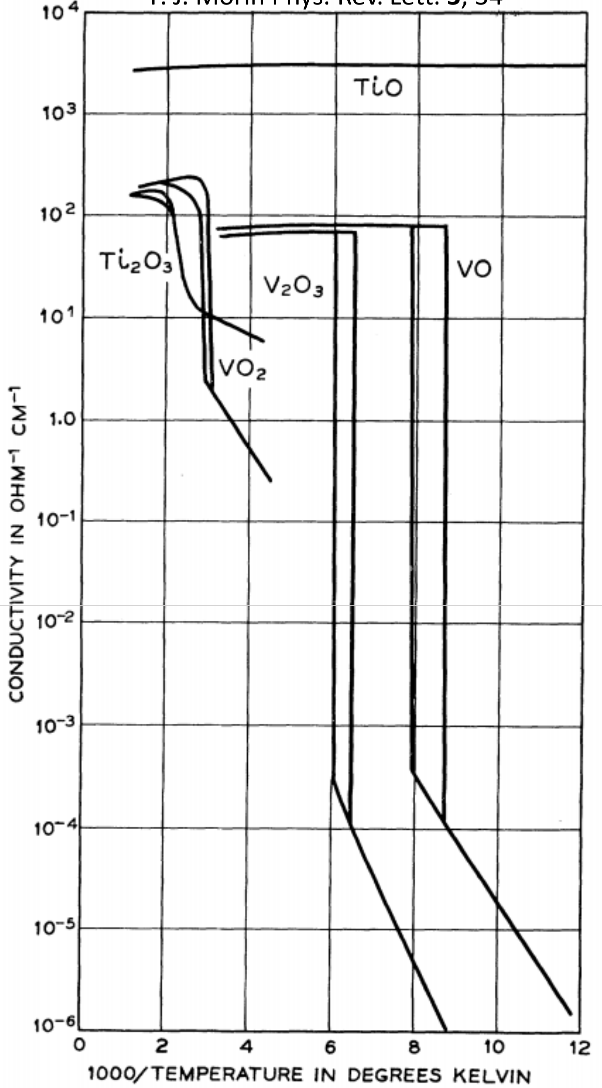
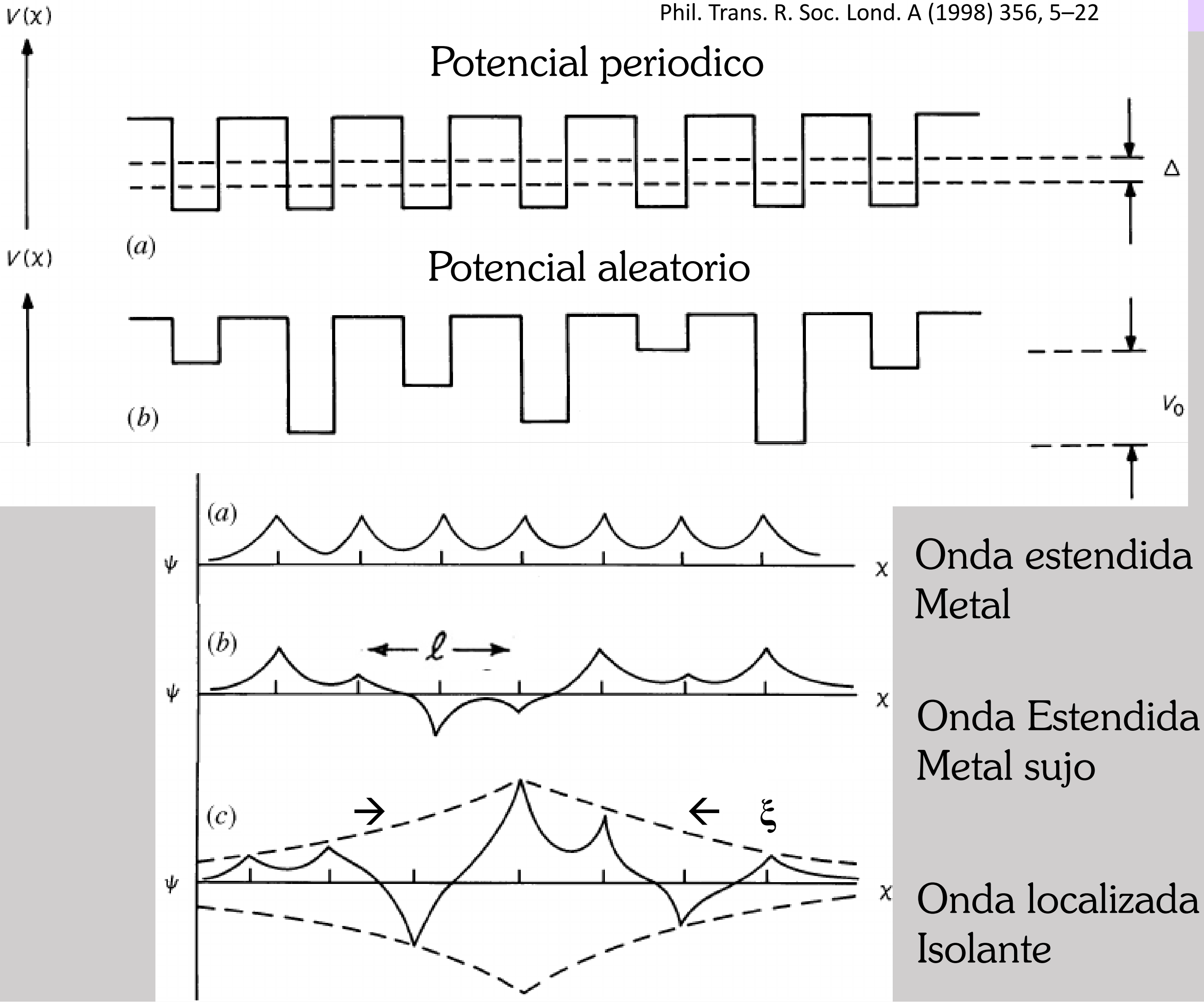


FIG. 1. Conductivity as a function of reciprocal temperature for the lower oxides of titanium and vanadium. Measurements were made along the [100] direction in VO, and along the *c* axis in $V_2O_3$ and $VO_2$

# Condução em metais na presença de desordem

Weak localization



Phil. Trans. R. Soc. Lond. A (1998) 356, 5–22

Potencial periodico

Potencial aleatorio

Onda estendida
Metal

Onda Estendida
Metal sujo

Onda localizada
Isolante

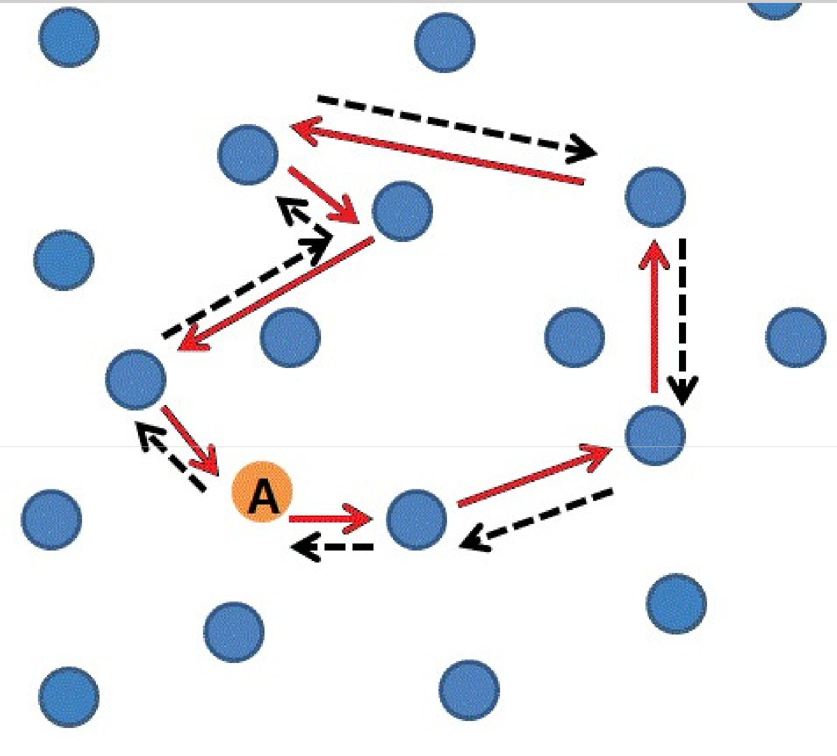
# Localização de Anderson

Amplitud de probabilidade de um eletron voltar para **A**

$$P = |\phi|^2 = \left|\sum_i \phi_i\right|^2$$

$$w = |A_1 + A_2|^2 = \underbrace{|A_1|^2 + |A_2|^2}_{w_{cl}} + \underbrace{2Re(A_1^* A_2)}_{w_{int}}$$

$$w = 4|A|^2 = 2w_{cl}$$



**Critério de Ioffe-Regel**

$$\ell \leq \frac{\lambda}{2\pi}$$

- Interferença – coherent Back scattering
- Efeito quântico de partículas individuais
- Dimensionalidade

# Experimentos

## Luz 3D

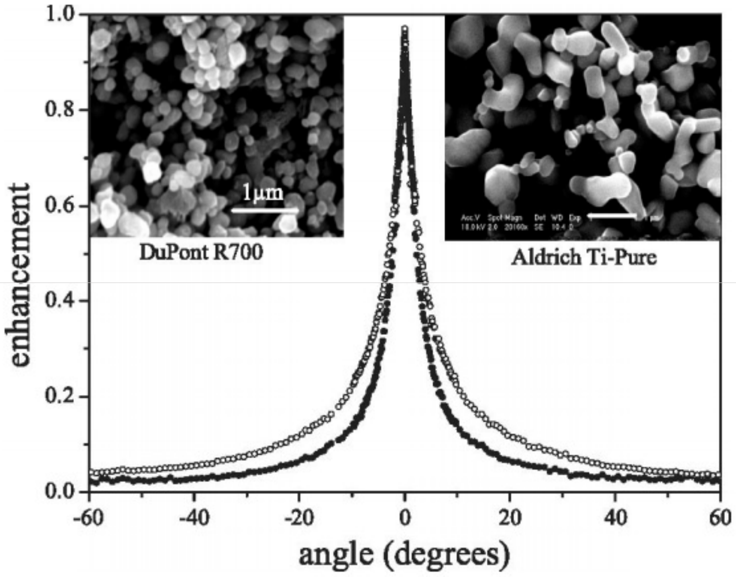


FIG. 1. Measurements of coherent backscattering for two different samples. Open symbols: R700 with an average particle diameter of 250 nm which yields $k\ell^* = 2.5$. Closed symbols: Ti-Pure with an average diameter of 540 nm and $k\ell^* = 6.3$. All measurements were done with circularly polarized light at a wavelength $2\pi/k = 590$ nm. The insets show electron micrographs of R700 and Ti-pure.

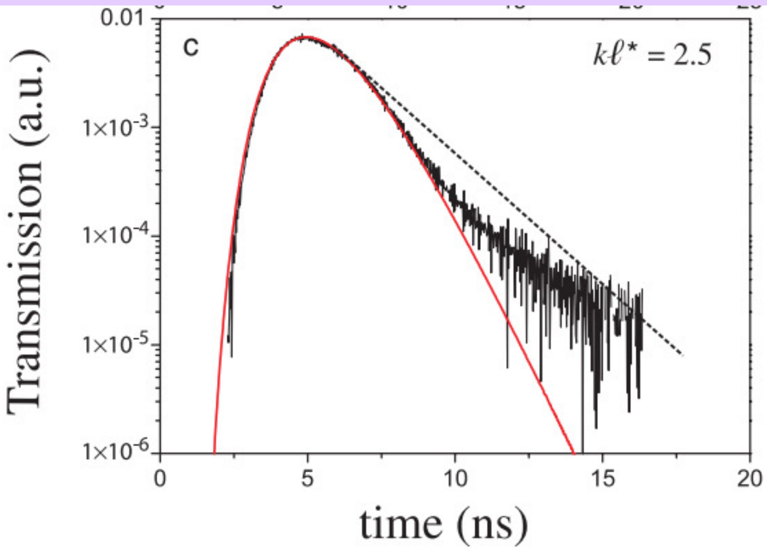


FIG. 2 (color online). Path length distributions from R700, R902, and Ti-Pure. The experimental results are compared to diffusion theory including absorption (full line). In (a), one can see that the data from Ti-Pure ($L = 2.5$ mm, $D = 22$ m$^2$/s, $\ell_a = 2600$ mm, $n = 1.28$) closely follow the diffusion fit, showing an exponential decay at long times. Part (c) in contrast shows strong deviations from the diffusion fit for R700 ($L = 1.48$ mm, $D = 15$ m$^2$/s, $\ell_a = 340$ mm, $n = 1.55$), with a clearly nonexponential decay at long times. These deviations can be explained by a time dependent diffusion coefficient in the sample. An intermediate case is shown in part (b) from R902 ($L = 1.51$ mm, $D = 13$ m$^2$/s, $\ell_a = 380$ mm, $n = 1.23$), with a value of $k\ell^* = 4.3$, where small deviations from the classical behavior can be observed. The respective values of the absorption length are indicated by the slope of the dashed lines.

# Experimentos

Cold Atoms 1D

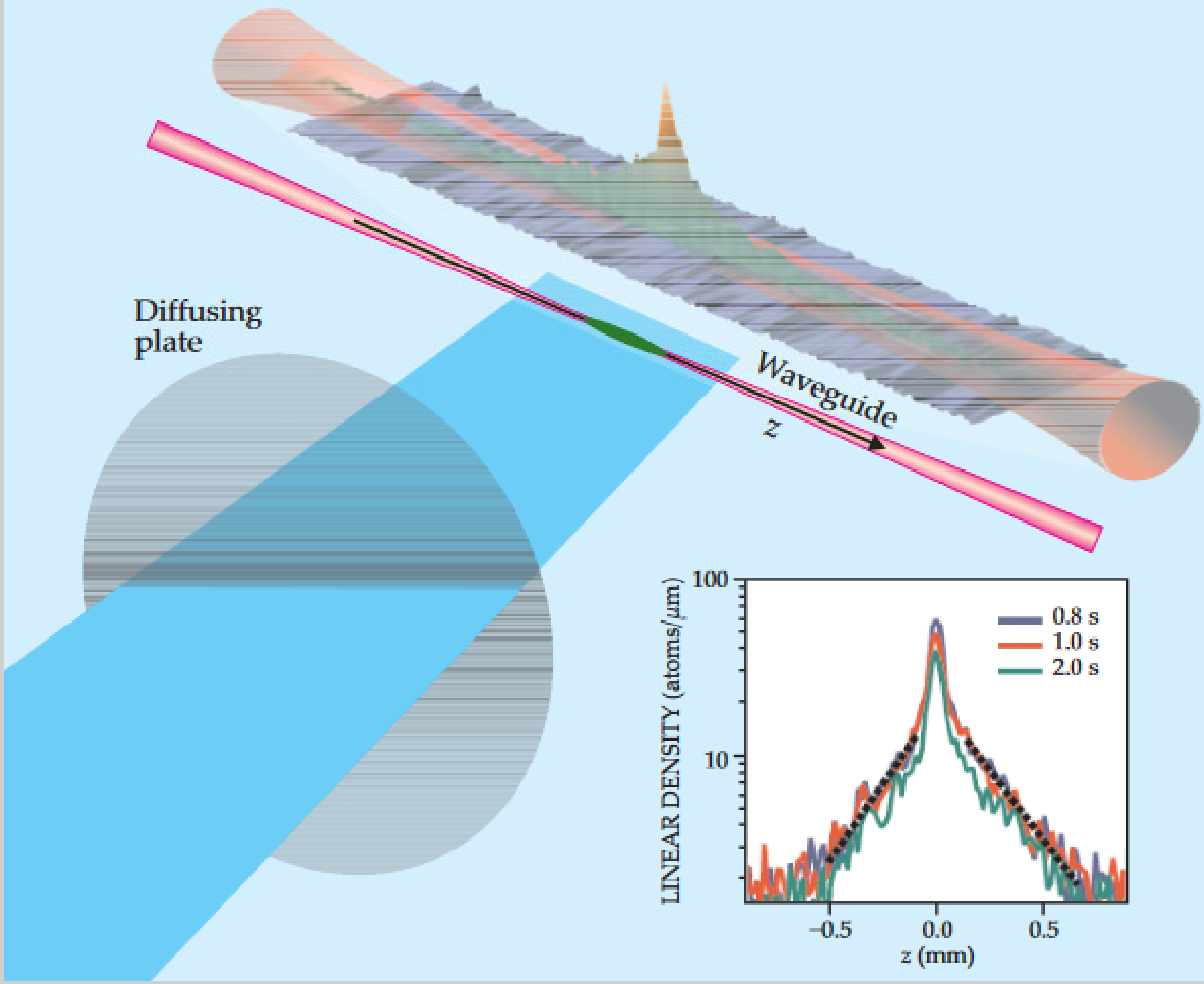


**Figure 3. Anderson localization** of ultracold atoms. The atoms are held by a matter waveguide that confines them transversely to the *z*-axis, but lets them travel freely along *z*. A laser beam passing through a thin aperture (elongated in the *z* direction) in a diffusive plate creates a disordered intensity pattern that varies rapidly along *z* and smoothly perpendicular to it. When a small Bose–Einstein condensate, initially confined along *z*, is released in the disordered potential, its expansion stops after about 0.5 s, after which a stationary density profile with exponentially decaying wings emerges. The semilog plots of the profiles at times 0.8 s, 1 s, and 2 s confirm the localization.

# Experimentos

Cold Atoms 2D

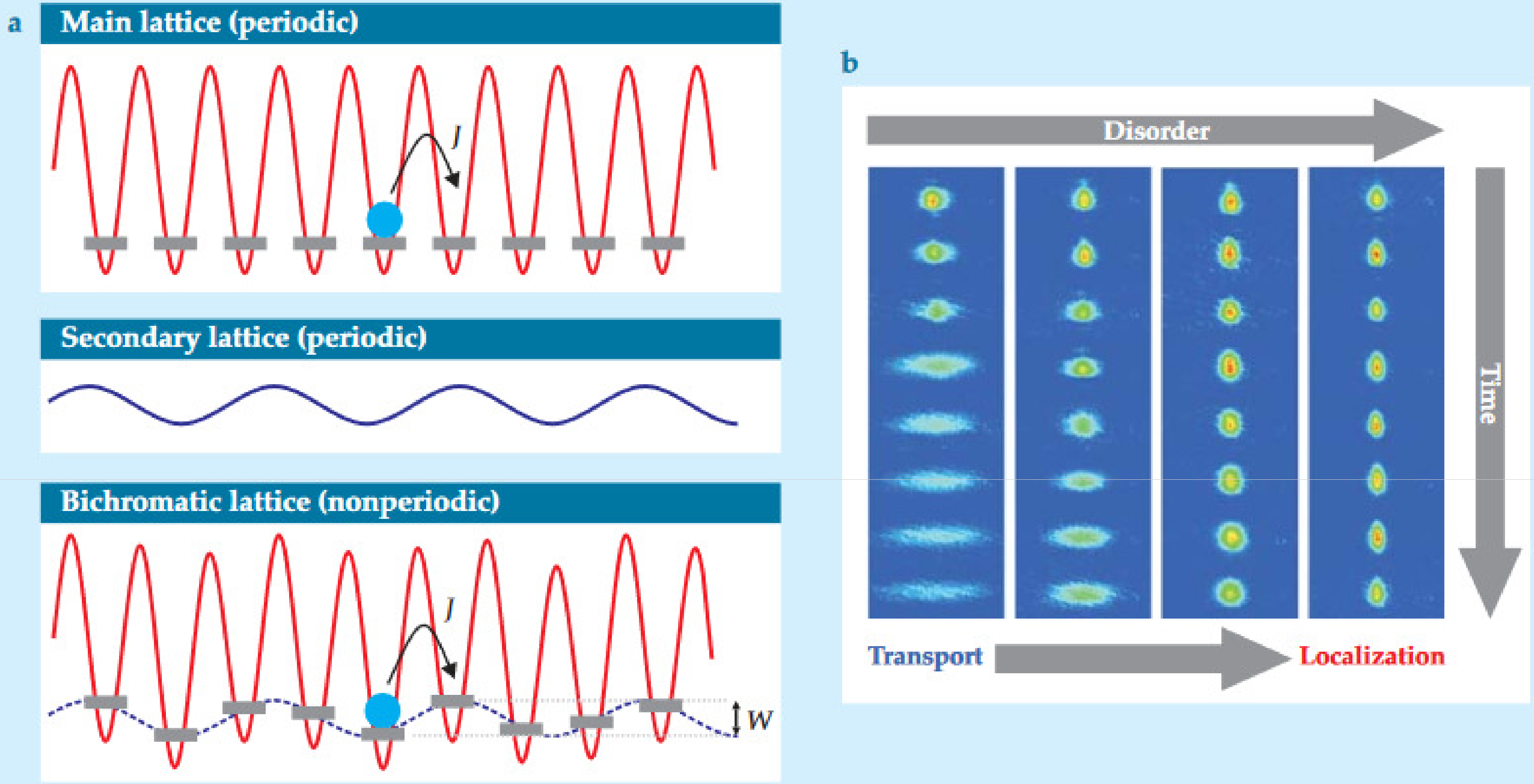


**Figure 4. A bichromatic lattice simulates solid-state model.** Loaded in a laser standing wave, an ultracold atom (blue) experiences a periodic potential. **(a)** The tunneling energy *J* can be controlled by changing the intensity of the main standing wave. A second, weaker optical lattice with an incommensurate spacing breaks the translational invariance and scrambles the site energies, as in the original solid-state model introduced by Philip Anderson. **(b)** Time-resolved images of almost noninteracting potassium-39 atoms. The atoms are first loaded into a few central sites of the bichromatic lattice and then observed diffusing into that nonperiodic structure. An increase in disorder leads to a decrease in diffusion and eventually its absence when the amount of disorder *W* becomes on the order of *J*.

# Experimentos

Cold
Atoms
3D

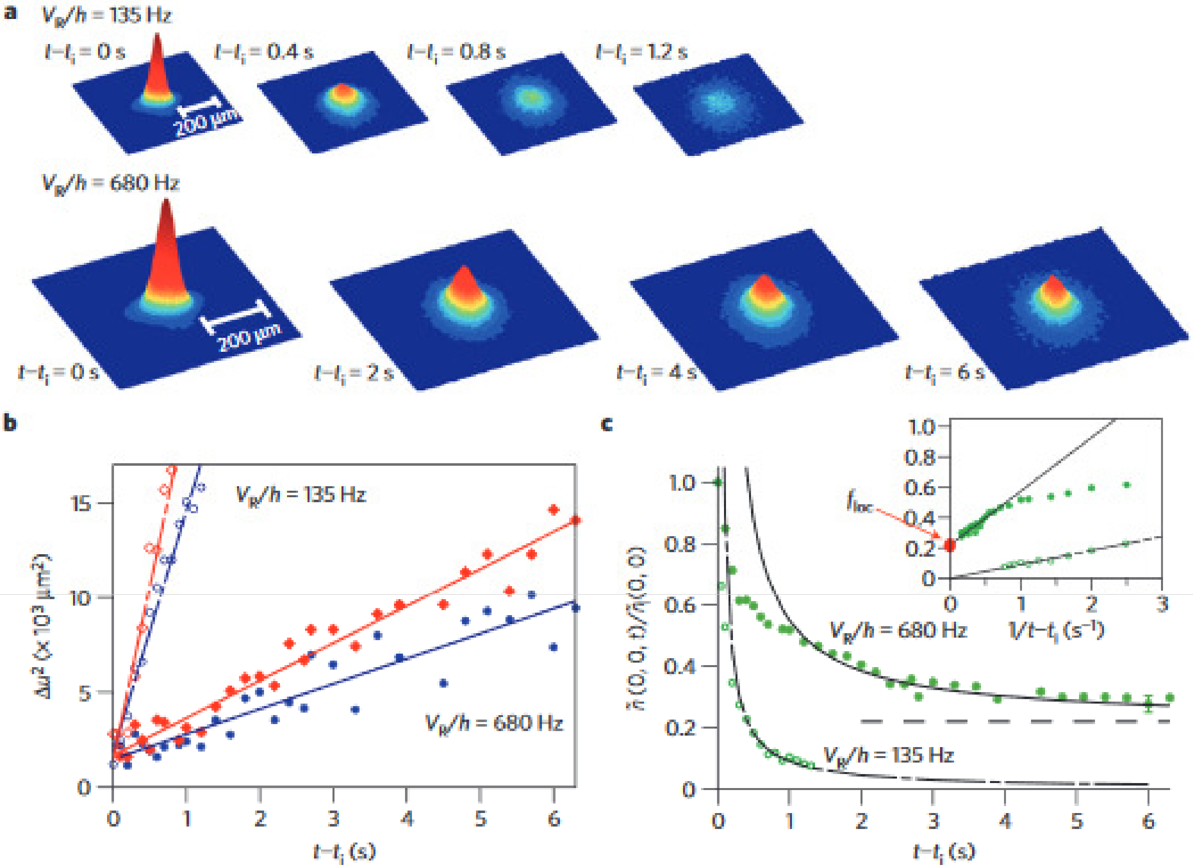


**Figure 2 | Evolution of the atomic cloud for two different amplitudes of the disorder. a,** Plots of the column density in the *y*–*z* plane, as observed by fluorescence imaging along the *x* axis (Fig. 1a) at various delays $t-t_i$ after application of the disorder. For a weak disorder ($V_R/h=135$ Hz), we observe an expansion leading to the disappearance of any observable atomic density for times larger than 1.2 s. For a strong disorder ($V_R/h=680$ Hz), the atomic cloud is still clearly visible after 6 s, and the profile shows a steady peak around the origin, superposed on a slowly expanding component. As shown in Fig. 2b, the expanding parts have a diffusive behaviour in both cases. **b,** Time evolution of the mean squared widths along *y* (blue circles) and *z* (red diamonds) of the column density profiles, and their fits by straight lines, yielding the diffusion coefficients along *y* and *z*. The anisotropy of the disorder, visible on Fig. 1b, is reflected on the diffusion coefficients. **c,** Time evolution of the column density at the centre (green circles). The black line is a fit by the function $A+B/(t-t_i)$, where the asympotic value $A$ is interpreted as the localized fraction $f_{loc}$ (see text). The inset shows the same data plotted as a function of $1/(t-t_i)$, fitted by the black straight line whose intercept on the left axis yields $f_{loc}$.

# Conclusões

•A localização de Anderson é um fenômeno ondulatório que poderia descrever transições nas propriedades de transporte de diferentes experiencias. Por ser um efeito de interferência de onda, as limitações de coerência de fase são um assunto importante de estudo neste contexto.

•A dependência da dimensionalidade do sistema com a localização de Anderson faz que este seja um fenômeno difícil de se observar para altas dimensões, fazendo diabolicamente complicada a tarefa de localização de elétrons num metal.

•Até agora, a localização de Anderson foi observada em muitos sistemas diferentes, sem dúvida. Por outro lado, a observação da transição de Anderson em si é uma tarefa muito mais desafiadora.

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# Fim

Obrigada ☺

# Transição de Anderson

## Teoria de dimensionamento -Scaling theory-

Comprimento de correlação

$$\xi \sim |E_c - E|^{-\nu}$$

Condutividade

$$\sigma \sim |E - E_c|^s$$

$$s = \nu(d-2)$$

Condutancia $g$
de um cubo d-dimensional

$$g(L) = \sigma L^{d-2}$$

Equação de Renormalização
de Grupo (RG)

$$\frac{d\ln(g)}{d\ln(L)} = \beta(g)$$

# Transição de Anderson